Multiple-wave lateral shearing interferometry for wave-front sensing

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Multiple-wave achromatic interferometric techniques are used to measure, with high accuracy and high transverse resolution, wave fronts of polychromatic light sources. The wave fronts to be measured are replicated by a diffraction grating into several copies interfering together, leading to an interference pattern. A CCD detector located in the vicinity of the grating records this interference pattern. Some of these wave-front sensors are able to resolve wave-front spatial frequencies 3 to 4 times higher than a conventional Shack–Hartmann technique using an equivalent CCD detector. Its dynamic is also much higher, 2 to 3 orders of magnitude. © 2005 Optical Society of America

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1. Introduction

Historically, the first wave-front sensor based on spatial phase sampling was probably the Scheiner disk,¹ in 1619. Since the invention of the laser, wave-front measurement techniques are part of the necessary tools for laser beam characterization. The need of a reference-free method explains why, until recently, the Hartmann test²,³ was the only widely used wave-front sensor for phase measurement, especially on large-aperture ( > 10 cm) laser systems. Hartmann-Shack⁴ wave-front sensors were originally developed in the 1970s. They soon became the devices of choice for astronomers looking for a wave-front sensor able to collect 100% of the photons. They were later adopted by laser physicists for their compactness and simple alignment procedure.

The Laboratoire pour l’Utilisation des Lasers Intenses (LULI), École Polytechnique, France, pioneered the use of achromatic multiple-wave lateral shearing interferometry (AMWLSI) for laser applications in the late 1990s.⁵–⁹ The principle of this technique relies on the generation of several replicas of the wave front to be evaluated using a grating. The resulting interferogram is then recorded onto a CCD retina, and the acquired image is finally processed using two-dimensional Fourier analysis to recover the phase with a high transverse and longitudinal resolution. For details regarding the principle of the technique, see the research of J. Primot.¹⁰,¹¹ As an example, an achromatic three-wave lateral shearing interferometer¹²,¹³ (ATWLSI) relies on the use of a transmission grating creating three replicas of the input wave front that are interfering together, resulting in a honeycomblike interference pattern.

The ATWLSI has, for instance, been used to evaluate with a high precision the nonlinear index $n_2$ of fused silica⁸ and also to create an on-axis focal line.¹⁴ All the work performed at LULI, especially in adaptive optics, relied on the ATWLSI.⁵–⁷,⁹,¹⁵,¹⁶ This wave-front sensor was also used extensively in this field at the Center for Ultrafast Optical Sciences at the University of Michigan.¹⁷,¹⁸ This was also the case for phase-conjugation experiments at Troitsk Institute of Innovative and Thermonuclear Research in Russia.¹⁹ The ATWLSI appeared indeed to be a good candidate for adaptive optics in lasers. For instance, in the framework of the European Union funded fifth Programme-cadre Communautaire de Recherche et Développement (Adaptool), it was cross evaluated by the five leading European laboratories using intense lasers. The limited transverse resolution of the Shack–Hartmann (SH) technique was then highlighted.²⁰

2. Achromatic Three-Wave Lateral Shearing Interferometer

The main components of an ATWLSI are described in this section (grating (Subsection 2.A) and mask (Sub-
section 2.B). The fundamental operations of the recovering algorithm are described, and the adjustable sensitivity leading to a high dynamic capability is explained.

A. Achromatic Three-Wave Lateral Shearing Interferometer Grating

Looking in the far field, the perfect grating would send exactly a third of the input energy in each summit of an equilateral triangle: \( \frac{\text{T}_{\text{id}}}{H^2} \), \( \frac{\text{T}}{H^2} \), \( \frac{\text{T}}{H^2} \). This field is a Fourier plane, where transverse space coordinates are \( (\mu, v) \); coordinates in the real space are \( (x, y) \).

Several algorithms can be used in order to optimize the grating profile. Most of them start with a Fourier transform of this energy distribution. The result gives a very good approximation of the ideal transmittance \( t_{\text{id}}(\mu, v) \) of the grating. The grating used in the first generation of ATWLSI is a pure phase object. This constraint implies dropping the amplitude term \( A(x, y) \) of the ideal transmittance and replacing it with a constant term. We are applying such a constraint in a Gerchberg–Saxton type of algorithm, which leads to the phase term \( \varphi(x, y) \) plotted in Fig. 1.

The first generation of ATWLSI is based on an approximated version of this phase distribution. A three-level-only phase plate. An ancestor of the ATWLSI was the three-wave lateral shearing interferometer. It was based on a splitting cube and a mirror to generate the three replicas by reflections on three different surfaces. But this device appears to be difficult to use with a polychromatic light source (a femtosecond laser pulse, for instance). Indeed, the interferfringe of the resulting interferogram was then dependant upon the wavelength. In order to make this three-wave lateral shearing interferometer achromatic, it was then necessary to use a grating. This technique is used, for instance, in the Mach–Zehnder grating.

The grating we describe here is a pure phase object working in transmission for a beam having a normal incidence. Its diffraction efficiency is optimum for a specific wavelength \( \lambda_0 \). The elementary cell is a group of three regular hexagons of respective height \( 0, \lambda_0/3, \) and \( 2\lambda_0/3 \), as shown in Fig. 2. This 2D diffractive element can also be seen as a superposition of three one-dimensional (1D) gratings whose groove pitch is equal to

\[
\text{pas} = \frac{3 \, \text{hex}}{2},
\]

where \( \text{hex} = 100 \, \mu\text{m} \) is the small dimension of a hexagon, as shown in Fig. 2.

Each of these three 1D gratings can be seen as “blazed” so that the energy of a wave at normal incidence would be almost entirely diffracted in orders \( m = +1 \) with an angle \( \beta \) satisfying

\[
\text{pas}[\sin(0) + \sin(\beta)] = m\lambda.
\]

Using Eq. (1) leads to

\[
\sin(\beta) = \frac{2\lambda}{3 \, \text{hex}}.
\]

Adding the fields of the three diffracted waves leads to an intensity distribution (the honeycomblike interferogram) from which the distance between two adjacent maxima can be extracted: the so-called “interfringe” (even if, in this case, fringes are dots):

\[
\text{if} = \frac{2\lambda}{3 \, \tan \beta}.
\]
Combining these last two equations, the interfringe expression becomes

$$i_f = \text{hex} \cos(\beta).$$  \hspace{1cm} (5)

The characteristic dimension (hex) of the grating is two orders of magnitude larger than the wavelength. The angle $\beta$ is consequently very small: dispersion properties are indeed very weak. An expansion around $\beta$ is then acceptable:

$$i_f = \text{hex}(1 + \frac{\beta^2}{2} + \ldots).$$  \hspace{1cm} (6)

Even the zero-order approximation is valid since the numerical calculation ($\lambda = 1.057 \, \mu m$ and hex = 100 \, \mu m) gives

$$i_f = \text{hex}(1 + 2.5 \times 10^{-5} + \ldots).$$  \hspace{1cm} (7)

We can then consider

$$i_f = \text{hex}.$$  \hspace{1cm} (8)

Gray-level lithography is used to process these gratings. The principle is described in Fig. 3. A variable-transmittance mask is used (after development) in order to create a spatial modulation of the thickness of a photosensitive resin deposited onto a silica substrate. Choosing a resin having the correct index of refraction may permit the process to be stopped after this step 1. The resulting plate must then be handled carefully because of the potential fragility of the resin. For applications where more robustness is required, one must pursue step 2: etching of the substrate. Then, we obtain a robust silica plate.

This overall process (using a pure phase grating and approximating the phase profile with three levels only) explains why the actual grating exhibits the experimental energy distribution shown in Fig. 4. Indeed, a nonnegligible amount of the input energy is transmitted by the grating in unwanted orders, creating then several other perturbing replicas.

We are currently working on a new version of gratings. Holography techniques should help obtain diffraction gratings having a much better efficiency.

B. Mask

Equation (8) shows that the interfringe is independent of $\lambda$, justifying the achromatic qualification of the ATWLSI. Nevertheless, this wave-front sensor is somehow limited in spectral bandwidth, as is shown in this subsection.

In order to overcome the issue of unwanted diffraction peaks, an order-selection plate has to be inserted in the common focal plane of the lenses of a telescope located between the grating and the sensor (Fig. 5). The widths of the holes are defined according to criteria linked to spectral overlap of high diffraction orders ($m \geq +1$). There are two consequences related to the use of this order-selection plate:
• The overall system stays achromatic, but its spectral bandwidth is limited (see Subsection 2.B.1).
• Phase measurements become limited to wave-front distortions, leading to a focal energy transverse distribution smaller than the hole aperture (see Subsection 2.B.2).

1. Spectral Bandwidth Limitation

The grating is working in the \( m = +1 \) order. It is recommended to avoid overlap between higher orders \( (m > +1) \) of the spectral component \( \lambda_{\text{min}} \) of the light source and the \( m = +1 \) order of the spectral component \( \lambda_{\text{max}} \). For a normal incidence at wavelength \( \lambda \), the “grating law” (2) gives

\[
\text{pas sin } \beta_m(\lambda) = m \lambda,
\]

where \( \beta_m(\lambda) \) is the value of angle \( \beta \) linked to the order \( m \) at wavelength \( \lambda \). Overlapping occurs when

\[
[\text{sin } \beta_m(\lambda_{\text{max}}) = \text{sin } \beta_m(\lambda_{\text{min}})] \Rightarrow [m \lambda_{\text{max}} = n \lambda_{\text{min}}].
\]

With \( m = +1 \) and \( n = +2 \), this equation is equivalent to

\[
\lambda_{\text{max}} = 2 \lambda_{\text{min}}.
\]

This equality defines \( \Delta \lambda_{\text{ATWLSI}} = [\lambda_{\text{min}}, \lambda_{\text{max}}] \), the spectral bandwidth of the ATWLSI. It is centered on \( \lambda_0 \) the wavelength for which the grating efficiency has been optimized:

\[
\lambda_0 = \frac{\lambda_{\text{min}} + \lambda_{\text{max}}}{2}.
\]

The expression of the spectral bandwidth is then

\[
\Delta \lambda_{\text{ATWLSI}} = [2\lambda_0/3, 4\lambda_0/3].
\]

The mask’s holes are defining this bandwidth. In order to show this, let us first recall that \( \beta \) is the incidence of the three diffracted waves arriving at the first lens of the telescope (lens L1 of focal length \( f_1 \)). According to Eqs. (4) and (8), it satisfies

\[
\tan \beta(\lambda) = \frac{2 \lambda}{3 \text{ hex}}.
\]

Let \( r(\lambda) \) be the radial coordinate, in the focal plane of L1, of a ray of wavelength \( \lambda \) intercepting this lens under incidence \( \beta \):

\[
r(\lambda) = f_1 \tan \beta(\lambda).
\]

Combining Eqs. (14) and (15),

\[
r(\lambda) = \frac{2 \lambda}{3 \text{ hex}} f_1.
\]

In order to satisfy Eq. (13), the holes of the select-order mask must then be limited by

\[
r_{\text{min}} = r(\lambda_{\text{min}}) = 2r_0/3,
\]

\[
r_{\text{max}} = r(\lambda_{\text{max}}) = 4r_0/3,
\]

where \( r_0 = r(\lambda_0) \).

2. Amplitude of Phase Distortions Measurement Limitation

Let us now consider the spreading of the energy distribution in the focal plane for a given wavelength \( \lambda \). Ideally, whatever the wavelength:

• The focal spot of the zero order should be masked (condition c1).
• The focal spot of the three first orders should not be truncated (condition c2).

Condition c1 guarantees that only three orders are interfering, whereas c2 means that the role of the mask is to select the useful orders of diffraction and not to act as a spatial filter, degrading the ability of the ATWLSI to detect highly distorted wave fronts.

The focal spots are identical for \( m = 0 \) and \( m = +1 \). For wavelength \( \lambda_0 \), c1 and c2 imply a double constraint to the mean radius \( r_{\text{foc}}(\lambda_0) \) of this focal spot:

\[
r_{\text{foc}}(\lambda_0) < r_{\text{min}}, \quad r_{\text{foc}}(\lambda_0) < \frac{r_{\text{max}} - r_{\text{min}}}{2} = \frac{r_{\text{min}}}{2}.
\]

Let us assume that the three beams incident on lens L1 carry a flat phase and a constant amplitude profile over a circular pupil of diameter \( \Theta \). Their focal energy distribution is then called an Airy pattern. Its size can be defined as the radius of the first dark ring of this distribution. At the central wavelength of the spectrum of the light source, its expression is

\[
r_{\text{Airy}}(\lambda_0) = \frac{1.22 \lambda_0 f_1}{2 \Theta}.
\]

These beams are said to be limited by diffraction. In other words, the transverse size of the focal energy distribution is limited by the diffraction of the wave on the edge of the \( \Theta \) diameter pupil.

If, now, the incident beam is carrying optical aberrations, its focal intensity distribution will spread. Let us call the limit of diffraction (LD) the ratio between \( r_{\text{foc}}(\lambda) \), the mean radius of the aberrant focal spot, and \( r_{\text{Airy}}(\lambda) \). A diffraction-limited beam is then characterized by an LD of unity, whereas LD is greater than one for an aberrant beam. This criterion, although widely used in optics, gives almost no information regarding the nature of the carried phase distortions, but it is a quick way to evaluate the transverse quality of a light beam.

The largest focal spot measurable at \( \lambda_0 \) with the ATWLSI can then be evaluated in terms of the limit of diffraction, \( \text{LD}_c \):
LD₀ = \frac{r_{\text{min}}}{r_{\text{Airy}(\lambda_0)}} = \frac{4}{9} \frac{\varnothing}{1.22 \text{ hex}} \quad (20)

For a \( \varnothing = 1 \text{ cm} \) diameter beam, this equation gives \( \text{LD}_0 = 36 \) for the optimal wavelength \( \lambda_0 \), and hex = 100 \( \mu \text{m} \). For the extreme wavelengths, \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \), the associated limits of diffraction \( \text{LD}_{\text{min}} \) and \( \text{LD}_{\text{max}} \), are obviously equal to zero because the center of their focal spot is located on the edge of the mask. It appears then pertinent to take into account the spectral bandwidth of the source in order to evaluate the limit of diffraction for \( \lambda_1 \) and \( \lambda_2 \):

\[
\text{LD}_1 = \frac{r(\lambda_1) - r_{\text{min}}}{r_{\text{Airy}(\lambda_1)}} = \text{LD}_0 \frac{(2\lambda_0 - 3\Delta\lambda)}{(2\lambda_0 - 3\Delta\lambda)},
\]

\[
\text{LD}_2 = \frac{r_{\text{max}} - r(\lambda_2)}{r_{\text{Airy}(\lambda_2)}} = \text{LD}_0 \frac{(2\lambda_0 - 3\Delta\lambda)}{(2\lambda_0 - 3\Delta\lambda)}.
\]

The above expressions are obtained while assuming the source to be centered at the wavelength \( \lambda_0 = (\lambda_1 + \lambda_2)/2 \). As an example, laser pulses used at LULI²⁶ satisfy \( \lambda_0 = 1.057 \mu \text{m}, \lambda_1 = 1.052 \mu \text{m}, \lambda_2 = 1.062 \mu \text{m}, \) and then \( \Delta\lambda = [\lambda_1, \lambda_2] = 10 \text{ nm} \), which leads to \( \text{LD}_1 = 35.7 \) and \( \text{LD}_2 = 35.3 \). The graph in Fig. 6 shows the evolution of both branches (21) and (22) of the curve \( \text{LD}(\lambda)/\text{LD}_0 \) in this specific case. Looking at this curve, it appears that the total dynamic of the ATWLSI is fully exploited only when the measurement is performed at the wavelength for which the device has been optimized. When the beam is very distorted (\( \text{LD}_0 > 36 \)) or is small in diameter, the three focal spots become large, and the mask starts to act as a spatial filter in a Fourier plane. The phase recovered from the interference pattern recorded on the CCD will then appear less aberrant than it really is. Figure 7 shows an ATWLSI with all its components.

C. Recovering Algorithm

A 2D Fourier analysis-based algorithm is used to extract the phase from the interference pattern recorded by the CCD. Figure 8 describes its structure:

Step 1. A first Fourier transform is performed onto the interferogram. The FT is a mathematical operation known for keeping symmetry properties. It is then not a surprise to observe a six-order symmetry pattern in the Fourier space: a zero-order harmonic surrounded by six first-order harmonics. Owing to the symmetry properties of the Fourier transform, only three of them carry different information.

Step 2. Any group of three first-order harmonics located on the vertices of an equilateral triangle is relevant to recover the phase. These harmonics are then windowed and extracted from the Fourier image.

Step 3. Each of the three small images resulting from the previous spectral windowing operation is then processed through an inverse Fourier transform operation. The three derivatives of the phase we are looking for are extracted from the resulting complex image.

Step 4. Finally, the three derivatives are integrated to recover the final phase.

In practice, several other image-processing operations (such as thresholding, masking, least-square projection, or Fourier integration) are required. The central harmonic can also be used in order to recover the amplitude transverse profile of the analyzed beam (Fig. 9). The overall process is here more straightforward.
The ATWLSI allows the recovery of three derivatives of the phase in three directions \((U_0, U_{120}, U_{-120})\) pointing at 120° from each other. It leads to interesting properties specific to ATWLSI:

First, the integration of three such derivatives gives a more accurate reconstruction of the phase than two derivatives at 90° from each other (as in a Shack–Hartmann, or for the achromatic quadri-wave lateral shearing interferometer (AQWLSI) described in Section 3) or only one derivative (as with a shearplate or a Mach–Zehnder interferometer).

Also, it can be demonstrated\(^{12,13}\) that the sum of the three derivatives obtained should mathematically cancel if the overall wave-front sensor (grating, CDD, . . .) were perfect:

\[
S(\mathbf{r}) = \nabla \varphi(\mathbf{r}) \cdot (U_0 + U_{120} + U_{-120}) = 0,
\]

where \(\mathbf{r}\) is the transverse coordinate and \(\varphi(\mathbf{r})\) is the phase we are looking to recover.

In practice, this sum is slightly different from zero owing to imperfections of the wave-front sensor itself. It can be shown\(^{12}\) that the mean value of \(S(\mathbf{r})\) over the pupil is directly linked to the phase root-mean square, \(\sigma:\)

\[
\langle S(\mathbf{r})^2 \rangle = 3\sigma^2.
\]

The ATWLSI possesses then the unique property of giving the phase profile together with an evaluation of the accuracy of this measurement.

D. Adjustable Sensitivity of the Achromatic Three-Wave Lateral Shearing Interferometer

A large dynamic range characterizes wave-front sensors of the AMWLSI family. Such interferometers are indeed able to detect phase distortions of several tens of waves but also of very small fractions of a wave \((\lambda/100)\). These sensors indeed have an adjustable sensitivity.

Let us call \(x\) and \(y\) the transverse coordinates of a light beam from which the phase \(\varphi(x, y)\) needs to be recovered. \(Oz\) is the direction of propagation. By using the diffractive grating described above, it is possible to duplicate \(\varphi\) in such a way that both replicas \(\varphi_{1,R}\) and \(\varphi_{2,R}\) are tilted with respect to each other by an angle \(\gamma\). These replicas are shown in Fig. 10 in the plane \(xOz\).

Their mean wave vectors \(\mathbf{k}_1\) and \(\mathbf{k}_2\) are not collinear but belong to \(xOz\). The tilt of \(\varphi_{1,R}\) and \(\varphi_{2,R}\) is

\[
\varphi_{1,R}(x, y) = \varphi(x, y) + \frac{\pi a}{\lambda} x,
\]

\[
\varphi_{2,R}(x, y) = \varphi(x, y) - \frac{\pi a}{\lambda} x,
\]

where \(a = \tan(\lambda/2)/2\). The plane where \(\varphi\) is duplicated is called the replication plane \(P_R\). This plane (the grating for the ATWLSI), when imaged onto a sensor, allows the recording of the interferogram \(I_R\):

\[
I_R(x, y) = 2I_0(x, y)(1 + \cos[\varphi_{1,R}(x, y) - \varphi_{2,R}(x, y)]).
\]

Combining Eqs. (25) and (26), we have

\[
I_R(x, y) = 2I_0(x, y) \left[1 + \cos \left(\frac{2\pi}{\lambda} ax\right)\right].
\]

This equation shows that, whatever the distortions of \(\varphi(x, y)\) are, \(I_R\) exhibits a regular fringe distribution characterized by an interference \(i = \lambda/\alpha\). Such an observation plane is called the “zero-sensitivity plane” of the interferometer, where all the information regarding \(\varphi(x, y)\) is lost.

After duplication, \(\varphi_1\) and \(\varphi_2\) will shear from each
other while propagating since they follow the directions of their respective wave vectors \(\mathbf{k}_1\) and \(\mathbf{k}_2\) (Fig. 10). If now, the observation is performed in a plane \(P_z\) located after \(P_R\) at a distance \(z\), the transverse shear between both replicas becomes

\[
dx = \frac{ax}{2},
\]

and the recorded interferogram \(I_z\) satisfies

\[
I_z(x, y) = 2I_0(x, y)[1 + \cos[\varphi_{1,z}(x, y) - \varphi_{2,z}(x, y)]],
\]

with this time,

\[
\varphi_{1,z}(x, y) = \varphi\left(x + \frac{dx}{2}, y\right) + \frac{\pi a}{\lambda}\left(x + \frac{dx}{2}\right),
\]

\[
\varphi_{2,z}(x, y) = \varphi\left(x - \frac{dx}{2}, y\right) - \frac{\pi a}{\lambda}\left(x - \frac{dx}{2}\right). \tag{30}
\]

Only the modulation \(M(x, y)\) of the intensity distribution \(I_z(x, y)\) carries useful information:

\[
M(x, y) = \cos[\varphi_{1,z}(x, y) - \varphi_{2,z}(x, y)]. \tag{31}
\]

Combining this equation with Eq. (30), one obtains

\[
M(x, y) = \cos\left[\frac{2\pi}{\lambda} ax + \varphi\left(x + \frac{dx}{2}, y\right) - \varphi\left(x - \frac{dx}{2}, y\right)\right]. \tag{32}
\]

Taking into account the dependence of \(dx\) with \(z\) given by Eq. (28), we obtain

\[
M(x, y) = \cos\left[\frac{2\pi}{\lambda} ax + \varphi\left(x + \frac{a}{4} z, y\right) - \varphi\left(x - \frac{a}{4} z, y\right)\right]. \tag{33}
\]

By introducing the interferinge, \(i = \lambda/a\), and by looking only at the \(x\) dimension perpendicular to the fringe, we have

\[
M(x) = \cos\left[\frac{2\pi x}{l} + \varphi(x + \lambda z/4i) - \varphi(x - \lambda z/4i}\right]. \tag{34}
\]

The first term \(2\pi x/l\) of the cosine argument defines the carrier modulation of the information contained in the second term, \(\Delta \varphi(x, z) = \varphi(x + \lambda z/4i) - \varphi(x - \lambda z/4i)\). If the transverse shear, \(dx = \lambda z/2i\), is sufficiently weak that the phase variation, \(\Delta \varphi(x, z)\), can be considered as linear, it appears then that \(I_z(x, y)\) allows recovering of the partial derivative in \(x\). Let us call \(\varphi'_z(x, z)\) the partial derivative at point \(x\):

\[
\varphi'_z(x, z) = \frac{\varphi(x_i + \lambda z/4i) - \varphi(x_i - \lambda z/4i)}{\lambda z/2i} \quad \text{for} \ z \neq 0.
\]

Let us evaluate the bandwidth associated to the measurement of \(\varphi'_z(x, z)\). Whatever the values of \(x\) and \(z\) are, one must have \(\Delta \varphi(x, z) < 2\pi\) in order to avoid any ambiguity in fringe attribution. Indeed, if \(\Delta \varphi(x, z) = 2\pi\), Eq. (34) then simplifies locally into \(M(x) = \cos[2\pi x_i/i]\); in other words, the carrier frequency is not suitable anymore. It becomes then for the maximum acceptable local tilt:

\[
\varphi_{\max}'(x, z) = \frac{\pi i}{\lambda z}. \tag{36}
\]

Let us assume that \(\Delta \varphi = \pi\) allows an optimal measurement (the fringe is shifting by half a period owing to the local gradient). It appears then that, in a point where this condition is satisfied, the optimally detectable local derivative \(\varphi'_z(x, z) = i2\pi/\lambda z\) is varying inversely with \(z\). This means that, when the plane of observation \(P_z\) is very far from \(P_R\), it is possible to detect a very weak local gradient with an optimal quality. Conversely, when \(P_z\) is getting closer (small \(z\)), it becomes then possible to detect very large gradients. In summary, this interferometer is characterized by a large dynamic of measurement owing to a sensitivity adjustable with \(z\) according to the amplitude of the wave-front distortions to be detected.

Figures 11 and 12 aim at experimentally showing this effect. An ATWLSI is used to measure the wave front of a laser beam going through a binary Fresnel-zone plate (250-nm height) used in plasma physics experiments for transversally smoothing an intense laser beam before focusing. The distance between the sensor plane (CCD) and the replication plane \(P_R\) is
adjusted from \( z/\text{hex} = 5 \) to \( z/\text{hex} = 60 \). Figure 11 shows a succession of 13 wave-front images obtained with the ATWLSI for these different values of \( z/\text{hex} \). hex is the characteristic dimension of the grating [see Eqs. (1) and (4)].

It is useful to compute the contrast of these wave-front images:

\[
\text{contrast} = \frac{(B - A)}{C}. \tag{37}
\]

The wave front under study is generated by a two-level-only phase plate, making such a quantity relevant here. \( C \) is the sensor dynamic; 256 gray levels in this case. The bottom right of Fig. 11 shows how, from the histogram of each of the 13 wave-front images, we extract the difference \( (B - A) \) between the two peaks associated with each of the two levels of the phase. The use of a perfect binary Fresnel-zone plate combined with an ideal sensor would give \( B - A = C \), leading to a contrast of unity.

By plotting the contrast versus \( z \) (Fig. 12), it appears that it increases with \( z \). It should be noted that the transverse resolution is simultaneously decreasing with \( z \) (less-fine defects are observed for high values of \( z \)).

This means that, for this specific 250-nm-height phase distortion, it is required to record the interferogram at a distance of 4 to 6 mm from the replication plane in order to have the best sensitivity (best contrast on the phase image). Above 6 mm, adjacent fringes start collapsing, making the interferogram analysis impossible.

3. Achromatic Quadri-Wave Lateral Shearing Interferometer

Primot et al.\textsuperscript{27} gave a demonstration of this device under the name of modified Hartmann mask. The terminology used here is voluntarily linked to the previously described ATWLSI. It is a semantic choice simply from the fact that the AQWLSI belongs to the common AMWLSI family. This interferometer relies on the use of four replicas, when three were required in the ATWLSI.

The exploration of this alternative solution is motivated by the lack of compactness of the ATWLSI. It is indeed shown in the previous section that, in order to obtain a distribution of the energy of the incident beam in only the three useful orders, an imaging and order-selection device must be added, making the ATWLSI more difficult to align and less compact. The AQWLSI is explored in order to overcome this issue.

A. Achromatic Quadri-Wave Lateral Shearing Interferometer Grating

We follow a procedure identical to the one described in Section 2: We observe energy distributions into the far-field plane, assumed here to be equivalent to a Fourier-space plane; ideally all the energy diffracted by the perfect grating should be located at the four vertices of a square [Eq. (38), where the tilde sign (\( \sim \)) symbolizes the Fourier transform]. The choice of a square in place of any other quadrilateral figure is mainly driven by symmetry reasons; for similar reasons, an equilateral triangle was chosen for the ATWLSI instead of an arbitrary triangle:

\[
\tilde{t}_\text{id}(v, \mu) = \delta(v - \frac{1}{a_0}, \mu - \frac{1}{a_0}) + \delta(v + \frac{1}{a_0}, \mu + \frac{1}{a_0}) + \delta(v + \frac{1}{a_0}, \mu - \frac{1}{a_0}) + \delta(v - \frac{1}{a_0}, \mu + \frac{1}{a_0}), \tag{38}
\]

\((v = \lambda/x, \mu = \lambda/y)\) are the Fourier-conjugated coordinates of \((x, y)\) in the real space, where \( \lambda \) is the wavelength. Going back to the real space through an inverse Fourier transform leads to the following simple expression for the ideal transmittance of the grating:

\[
t_{\text{id}}(x, y) = \frac{1}{2} \left\{ \cos \left[ \frac{2\pi(x + y)}{a_0} \right] + \cos \left[ \frac{2\pi(x - y)}{a_0} \right] \right\}. \tag{39}
\]

Let the ideal transmittance be reformulated as follows:

\[
t_{\text{id}}(x, y) = \text{sign}[t_{\text{id}}(x, y)] \text{abs}[t_{\text{id}}(x, y)], \tag{40}
\]

where the sign and abs functions are, respectively, the sign and the absolute value of the transmittance. Experimentally obtaining the sign function is relatively simple. Indeed, the \( t_{\text{sign}}(x, y) = \text{sign}[t_{\text{id}}(x, y)] \) distribution can be obtained with a phase chessboard having two altitudes of 0 and \( \pi \). Indeed \( \exp(\text{i}\pi) = -1 \), whereas \( \exp(0) = 1 \) (with \( i^2 = -1 \)). The chessboard period needs to be equal to \( p = a_0/\sqrt{2} \). A pure phase plate (made of fused silica, for instance) modulated according these rules is not difficult to process. However, this is not the case for the \( t_{\text{abs}}(x, y) = \text{abs}[t_{\text{id}}(x, y)] \) distribution. Creating a plate with a...
transmission varying according to the semisinusoidal arches of this distribution is more difficult, especially if the process used to do so must avoid the creation of any unwanted phase modulation: we are here indeed looking for a pure amplitude modulation for $t_{\text{abs}}(x, y)$.

As for the ATWLSI, approximating the ideal transmittance also appears to be required. The idea is to generate $t_{\text{abs}}(x, y)$ with only a two-level amplitude distribution. Figure 13 describes this operation. The resulting amplitude part of the grating appears to be a simple Hartmann plate with holes of square aperture  $a$ distributed over a Cartesian grid of period $p$. Equation (41) gives the expression of such an approximated version of $t_{\text{abs}}(x, y)$:

$$t_{\text{abs}}^{\text{approx}}(x, y) = \Pi_{a, a}(x, y) \otimes \psi_{a, a}(x) \otimes \psi_{a, a}(y),$$  \hspace{2cm} (41)

where $\psi_{a}(x)$ is the “comb” function, i.e., a succession of Dirac $\delta(x)$ functions regularly spaced with a $p$ period, and $\Pi_{a, a}(x, y)$ is a “gate” function having the same transverse aperture $a$ in both directions.

We show that by choosing carefully the ratio $a/p$, it is possible to select in which order we want the energy to be diffracted. In order to make it easier to visualize graphically, (Fig. 14) the individual contributing effect of $t_{\text{sign}}(x, y)$ and $t_{\text{abs}}(x, y)$, let us continue the demonstration with a 1D approach. We have then

$$t_{\text{abs}}^{\text{approx}}(x) = \Pi_{a}(x) \otimes \psi_{a}(x)$$  \hspace{2cm} (42)

and for its Fourier transform

$$\tilde{t}_{\text{abs}}^{\text{approx}}(v) = \frac{\sin(\pi v a)}{\pi v a} \otimes \psi_{1/p}(v).$$  \hspace{2cm} (43)

The grating is made of the combination of the Hartmann plate and the phase chessboard having the same period $p$. We have then the following 1D expression for the transmittance of our complete grating and its Fourier transform:

$$t^{\text{approx}}(x) = \Pi_{a}(x) \otimes \left[ \psi_{a}(x) \exp(i \pi x / p) \right]$$  \hspace{2cm} (44)

$$\tilde{t}^{\text{approx}}(v) = \frac{\sin(\pi v a)}{\pi v a} \otimes \delta \left( v - \frac{1}{2p} \right).$$  \hspace{2cm} (45)

Comparing Eqs. (43) and (45) reveals that the effect of the phase chessboard is simply to translate by $1/2p$ the Dirac comb of period $1/p$. As a consequence, there is no more energy observed in the zero order (arrow number 3 in Fig. 14). This is already a major improvement from the grating used for the ATWLSI. But it is still possible to do better.

Indeed, by adjusting the respective values of the Hartmann plate aperture $a$ and the period $p$, we can suppress several other unwanted diffraction orders. The idea is to make the orders that are to be canceled coincide with the zeros of the $\sin(\pi v a)/(\pi v a)$ function. Figure 14 describes this effect. A simple geometrical argument shows that the following condition satisfies the cancellation of orders labeled 1 and 4:

$$a = 2p/3.$$  \hspace{2cm} (46)

Let us now visualize the effect of Eq. (46) in a 2D space. Figure 15 details what can be observed in the Fourier plane, whereas Fig. 16 gives a picture of the real plane, i.e., the plane where the CCD sensor is located. Experimentally, we observe, in the Fourier plane, the distribution shown in Fig. 17. Such a dif-

Fig. 13. Top-right sketch is the phase part of the AQWLSI grating, i.e., a phase chessboard equivalent to sign($t_{\text{abs}}(x, y)$). The top-left sketch is the amplitude part, i.e., an approximation of abs($t_{\text{abs}}(x, y)$). This part appears to be a simple Hartmann plate with holes of square aperture $a$ distributed over a Cartesian grid of period $p$. The bottom curves show the lineout of abs($t_{\text{abs}}(x, y)$) (semisinusoidal arches curve) and its approximation (rectangular curve).

Fig. 14. Energy distribution $t^{\text{approx}}(v)$ given by a Hartmann plate of aperture $a$ and period $p$ is shown in this graph: The energy is located at the summit of each bold arrow (five of them are represented here with numbers in square boxes). The energy distribution $\tilde{t}^{\text{approx}}(v)$ given by the same Hartmann plate combined with a phase chessboard of period $p$ is also shown: The effect of the phase chessboard is to translate the previous set of arrows (i.e., energy) by $1/2p$ to give the dashed arrows (the new positions are labeled ① to ⑤). Arrows ① and ⑥ are replaced by two stars at the place where they should be located. There is in fact no energy at these positions owing to the presence of the zeros ($\pm 1/a$) of the $\sin(\pi v a)/(\pi v a)$ envelope. In order to achieve the cancellation of this two orders, the condition $a = 2p/3$ must be satisfied.
fraction pattern allows using the grating simply as the microlens array of a SH wave-front sensor: indeed, it just needs to be located in front of a CCD without the need for any extra image relay optics. The overall AQWLSI interferometer is much more compact than the ATWLSI. The angle $\alpha/2$ under which the four orders are being diffracted satisfies

$$\sqrt{2}p \sin\left(\frac{\alpha}{2}\right) = \lambda. \quad (47)$$

The grating used to produce the interferogram of Fig. 16 is characterized by $a = 26 \, \mu\text{m}$ and $p$

Fig. 15. Looking in the far field (for instance, in the focal plane of a lens located after the grating), we have access to the energy distribution in different orders of diffraction. In the left and right images, we see that there is no energy in the zero order (thanks to the phase chessboard). Let us call the four first orders the four peaks located the closest to the center of the field of this spectrum (all four belong to the same ring centered in the center of the field; ring one is in the middle image). In the left image, we can see eight peaks located in a second ring surrounding these four peaks ($\alpha \neq 2p/3$). These orders of diffraction vanish in the right picture ($\alpha = 2p/3$).

Fig. 16. Insert shows an AQWLSI grating, a Hartmann mask modified by the adjunction of a phase chessboard; this is why this wave-front sensor is also called modified Hartmann mask. The main picture is an interference pattern obtained with an AQWLSI based on a grating characterized by $a = 26 \, \mu\text{m}$ and $p = 39 \, \mu\text{m}$.

$= 39 \, \mu\text{m}$. It consequently gives a value of $36.26 \, \text{mrad}$ for $\alpha$ at $1 \, \mu\text{m}$. This angle is very small; this grating (like the ATWLSI's one) is then not a very strong dispersing element.

B. Transverse Resolution of the Achromatic Quadri-Wave Lateral Shearing Interferometer

Figure 18 gives experimental measurements of a distorted wave front recorded with an AQWLSI. The comparative analysis of the transverse resolution is good but does not appear very satisfying with respect to what was achieved with the ATWLSI. A reason is the fact that the AQWLSI allows recovery of only two derivatives, whereas the ATWLSI gives one more. The reconstruction accuracy is therefore degraded. It is also important to take into account the distance $z$ between the replication plane and the sensor. In the case of the ATWLSI, the grating plane is imaged onto

Fig. 17. Energy distribution recorded in the focal plane of a lens located right after the grating. More than $87\%$ of the incident energy is distributed in the four first orders. Less than a percent is present in the zero order, while no energy can be recorded in ring 2. The remaining energy distributed over a large number of higher orders does not affect the measure.

Fig. 18. Interferograms and (inserts) recovered laser phase obtained with an AQWLSI. The laser wave front was modulated using a liquid-crystal optical valve. The pupil is $\sim 5 \, \text{mm}$ and distortions amplitude is $\sim 1 \, \text{wave}$.
the CCD with a relay telescope, creating a virtual replication plane. It is consequently possible to adjust continuously the distance \( z \) from 0 to several centimeters. On the other hand, for the AQWLSI, the replication plane is real and equivalent to the grating plane; consequently, mechanical constraints do not permit getting too close to the sensor (\( z > 0 \)).

Figure 19 helps understand that when \( z \) increases, the transverse resolution degrades. The principle of AMWLSI interferometry is based on the transverse shear of replicas of a phase to be measured (Subsection 2.D). The larger this shear is, the worse the transverse resolution becomes. The transverse resolution is the sum of several factors (such as \( z \), grating, CCD, and wave front under study). The dependence in \( z \) is the distance \( D_z \) satisfying

\[
D_z(z) = \frac{\alpha}{\sqrt{2}} z = \alpha' z.
\]

We have then

\[
\alpha' = 25.64 \text{ mrad}.
\]

In order to check this dependence, we evaluated (Fig. 20) the variation of the measured width of a phase step using a phase plate of 150-nm height. The resulting slope (25.62 mrad) is in almost perfect agreement with the expected value (49). Nevertheless, the retrieved step width is 10 to 20 times larger than for the real object. The width of this step is equal to \( \sim 30 \mu m \).

4. Conclusion: Comparative Performances of Shack–Hartmann and Achromatic Multiple-Wave Lateral Shearing Interferometers

Coming from astronomical applications, the Shack–Hartmann wave-front sensor is nowadays widely used by laser users as well. The increased computing power of computers (fast and efficient real-time 2D Fourier analysis of large images) combined with performances of microraster EktiF of optical materials allowed the emergence of the AMWLSI wave-front sensor family. These techniques appear as an alternative to SH.

Both techniques are almost achromatic and thus suitable for recording wave fronts of laser short pulses. The AMWLSI sensors have a better transverse resolution for some applications and have a much larger dynamic measurement ability. Also, the cost of both techniques differs sharply. Comparing a SH and an AQWLSI, both require the same kind of components:

- the grating and the microlens array (similar cost);
- a computer and acquisition card (identical cost);
- a CCD or complementary metal-oxide semiconductor camera.

The cost of either technique is very dependent on this last component. Indeed, as is described in Subsection 4.A, the SH sensor requires many more pixels as well as very good dynamic (more than 8 bits) in order to correctly record the focal spots’ distribution. A digital CCD is usually required. On the other hand, the simple sinusoidal distribution to be recorded for the AMWLSI can be achieved with a standard low-cost analog camera.

A. Transverse Resolution

The transverse resolution of a SH is limited by the number of microlenses across the field of detection. For instance, a standard CCD (\( 500 \times 500 \) square pixels of 10 \( \mu m \)) and a standard SH sampling (16 \( \times \) 16 pixels per microlens aperture) leads to a transverse resolution of 160 \( \mu m \). Such a relatively high sampling rate (16 points per subaperture) is driven by the nature of the energy distribution to be sampled by the CCD grid. Indeed, for a SH, it is required to sample a sinc function, since, usually, microlenses have a square ap-
In the case of the AMWLSI, the energy distribution to be sampled is an interferogram having a sinusoidal modulation. Efficiently sampling a sinusoid requires a very limited number of points. Indeed, it requires fewer CDD pixels to sample a single-spatial-frequency signal such as a sine function rather than a sinc function carrying multiple spatial frequencies. For both ATWLSI and AQWLSI, the sampling rate is between three and four CCD pixels per fringe. In theory, the transverse resolution of AMWLSI is close to the distance between fringes, i.e., 30 to 40 μm for the same standard CCD. AMWLSI wave-front sensors thus have the potential of being almost four times better at resolving than SH sensors. Experimentally, this has been verified for AMWLSI for slow-varying wave-front distortions (fifth-order geometrical aberrations, for instance). As soon as the wave front to be measured starts exhibiting large gradients (such as those shown in Fig. 8 and insert of Fig. 20), the behavior of ATWLSI and AQWLSI differs significantly: ATWLSI remains very accurate, whereas AQWLSI shows very strong degradation in transverse accuracy as for a SH. Figure 20 shows indeed that the retrieved step width is 10 to 20 times larger than for the real object. This undermines seriously the capacity of AQWLSI to resolve fast-varying transverse structures. The three-derivatives-based ATWLSI sensor appears consequently very much more suited for accurately measuring such wave-front distortions than its two-derivatives-based counterpart, namely, SH and AQWLSI.

B. Dynamic Range

In order to evaluate the maximum gradient measurement dynamic range for both techniques, we look at a local level, i.e., across a microlens of aperture $\varnothing_i$ and focal length $f$ for a SH and across a fringe of width $i$ and at a distance $z$ for the AMWLSI. The maximum local gradient detectable with a SH is

$$\varphi_{\text{max}}(x_i, f) = \frac{2\pi \varnothing_i / 2}{f}.$$  \hspace{1cm} (50)

Indeed, a wave carrying a local gradient above this value would not be correctly evaluated since the energy focused by this specific microlens would not only be illuminating its own subaperture of pixels (16 \times 16) but also the neighboring one.

For an AMWLSI, the maximum local gradient detectable exhibits a very similar expression (z, being the distance between the detector and the grating):

$$\varphi_{\text{max}}'(x_i, z) = \frac{4\pi i}{\lambda z}.$$  \hspace{1cm} (51)

In fact, if we use (as was suggested in the previous subsection) $\varnothing_i = 160 \mu m$ and $i = 40 \mu m$, we see that $f$ and $z$ have exactly the same role. For a short focal length $f$ (for SH) or a short distance $z$ between detector and grating (for AMWLSI), the strongest gradients can be detected. A standard value for $f$ is $\sim 10$ mm. For an AMWLSI, it is possible to make $z$ varying between 0.1 and 100 mm, allowing then this wave-front sensor to provide a dynamic range 3 orders of magnitude higher than that of a Shack–Hartmann wave-front sensor. Table 1 summarizes the respective performances of these wave-front sensors.

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References


